

CP violation and Leptogenesis in models with Minimal Lepton Flavour Violation

Vincenzo Cirigliano^{a,b}, Gino Isidori^c, Valentina Porretti^d

^a *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

^b *California Institute of Technology, Pasadena, CA 91125, USA*

^c *INFN, Laboratori Nazionali di Frascati, Via E. Fermi 40, I-00044 Frascati, Italy*

^d *Dip. di Fisica, Univ. di Roma Tre, Via della Vasca Navale 84, I-00146 Roma, Italy*

Abstract

We investigate the viability of leptogenesis in models with three heavy right-handed neutrinos, where the charged-lepton and the neutrino Yukawa couplings are the only irreducible sources of lepton-flavour symmetry breaking (Minimal Lepton Flavour Violation hypothesis). We show that in this framework a specific type of resonant leptogenesis can be successfully accomplished. For natural values of the free parameters, this mechanism requires a high right-handed neutrino mass scale ($M_\nu \gtrsim 10^{12}$ GeV). By means of a general effective field theory approach, we analyse the impact of the CP violating phases responsible for leptogenesis on the low-energy FCNC observables and derive bounds on the scale of flavour violating new physics interactions. As a result of the high value of the scale of total lepton-number violation, in this class of models the $\mu \rightarrow e\gamma$ decay is expected to be close to the present exclusion limit (under the additional assumption of new particles carrying lepton flavour at the TeV scale).

1 Introduction

All extensions of the Standard Model (SM) with new degrees of freedom at the TeV scale carrying flavour quantum numbers have to face the severe constraints implied by low energy Flavour Changing Neutral Current (FCNC) transitions. An economical and elegant solution to this *flavour problem* is provided by the Minimal Flavour Violation (MFV) hypothesis, namely by the assumption that the irreducible sources of flavour symmetry breaking are minimally linked to the fermion mass matrices observed at low energy. On the one hand, the MFV hypothesis guarantees a suppression of FCNC rates

to a level consistent with experimental constraints without resorting to unnaturally high scales of new physics. On the other hand, this hypothesis provides a predictive and falsifiable framework that links the possible deviations from the SM in FCNC transitions to the measured fermion spectrum and mixing angles.

The MFV hypothesis has a straightforward and unique realization in the quark sector [1, 2, 3]: the SM Yukawa couplings are the only sources of breaking of the $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ quark-flavour symmetry. The extension of the MFV hypothesis to the lepton sector (Minimal Lepton Flavour Violation, MLFV) is less straightforward: a proposal based on the assumption that the breaking of total lepton number and lepton flavour are decoupled in the underlying theory has recently been presented in Ref. [4] and further analysed in Ref. [5]. The requirement of minimality and predictivity has lead to the identification of two independent MLFV scenarios [4], characterized by the different status assigned to the effective Majorana mass matrix m_ν^{eff} appearing as coefficient of the $|\Delta L| = 2$ dimension-five operator in the low energy effective theory [6].

In the truly minimal case (dubbed *minimal field content*), m_ν^{eff} , together with the charged-lepton Yukawa coupling λ_e , are assumed to be the only irreducible sources of breaking of $SU(3)_{L_L} \times SU(3)_{e_R}$ (the lepton-flavour symmetry of the low-energy theory). One of the consequences of this hypothesis is the fact that the only CP-violating phases involved in low-energy FCNC observables are those contained in the lepton mixing matrix U_{PMNS} . As a result, within this scenario it is not possible to address potential correlations between low-energy observables and high-energy phenomena such as leptogenesis [7]. The viability of leptogenesis crucially depends on the UV details of the model (see for instance Refs. [8, 9, 10]), which are beyond the control of the effective field theory approach.

In many realistic extensions of the SM, the flavour structure of the theory is modified by the presence of heavy right-handed neutrinos. For this reason, a second scenario (dubbed *extended field content*), with heavy right-handed neutrinos and a larger lepton-flavour symmetry group, $SU(3)_{L_L} \times SU(3)_{e_R} \times O(3)_{\nu_R}$, has also been considered. In this extended scenario, the most natural and economical choice about the symmetry-breaking terms is the identification of the two Yukawa couplings, λ_ν and λ_e , as the only irreducible symmetry-breaking structures, in close analogy with the quark sector. In this context, $m_\nu^{\text{eff}} \sim \lambda_\nu^T \lambda_\nu$ and the lepton-number-breaking mass term of the heavy right-handed neutrinos is flavour-blind (up to Yukawa-induced corrections).

In the extended MLFV scenario the assumption about the irreducible sources of lepton-flavour breaking does involve the high energy sector of the theory. In particular, the symmetry principle provides significant constraints on the amount of CP violation in the decays of right-handed neutrinos, and thus on leptogenesis. This could allow to establish some links between low-energy FCNC observables and leptogenesis. The investigation of these links is the main purpose of this work. We address in particular the following questions:

- Is leptogenesis viable in models where the only sources of flavour breaking are proportional to the charged-lepton and neutrino Yukawa matrices λ_e and λ_ν ? In other words: can one build nontrivial CP violating re-phasing invariants using only $\lambda_{\nu,e}$, and do they contribute to the CP asymmetries relevant for leptogenesis? This question has implications beyond the MLFV framework and our findings provide

a significant extension to the existing statements in the literature [11].

- In presence of CP violation, what is the flavour structure of the effective FCNC couplings? Lifting the technical assumption of CP conservation which was previously invoked to gain predictive power [4, 5], is the framework still predictive?
- Does the requirement of successful leptogenesis reduce the large uncertainty on the scale of lepton-number violation? Does this help to reduce the overall uncertainty in the predictions of low-energy FCNC rates?

In Section 2, after introducing the basic structure of the model, we show that the first of the above questions has a positive answer. We indeed find nontrivial re-phasing invariants that are in one-to-one correspondence with the CP asymmetries in the decays of the quasi-degenerate heavy Majorana neutrinos. Therefore, leptogenesis is in principle viable within this framework.

Next we perform a numerical study of leptogenesis (Sect. 3) and analyse the implications for charged-lepton FCNC processes (Sect. 4). We find that leptogenesis is also phenomenologically acceptable within this framework. In general, the presence of new CP-violating (CPV) phases leads to non-trivial modifications of the pattern of FCNC rates obtained in the CP conserving limit. However, we also find that there is an interesting regime of small CPV phases where the FCNC pattern is dictated again by the neutrino oscillation parameters. Within this regime, the flavour structure of the effective FCNC couplings receives small corrections with respect to the CP conserving case and the framework is particularly predictive.

The most interesting outcome of the numerical study of leptogenesis in the MLFV framework is an approximate lower bound on the right-handed neutrino mass: $M_\nu \gtrsim 10^{12}$ GeV, for natural values of the free parameters. This allows us to address the third question: we find that the requirement of both successful leptogenesis and FCNC rates in agreement with experiments leads to non-trivial constraints on the overall scales of new physics (more precisely the scales of lepton-number and lepton-flavour breaking). In particular, we strengthen the conclusion of Ref. [4] that within MLFV models the $\mu \rightarrow e\gamma$ decay is expected to be within the reach of the MEG experiment [12].

2 CP violation in models with MLFV and heavy ν_R

2.1 Framework

The *extended field content* MLFV scenario postulates the existence –beyond the Standard Model (SM) degrees of freedom– of three right-handed neutrino fields singlets under the SM gauge group. The right-handed neutrino mass (the only source of $U(1)_{\text{LN}}$ breaking) is flavour-blind and its scale is large compared to the electroweak symmetry breaking scale ($M_\nu \gg v$):

$$\mathcal{L}_{(0)} \supset \mathcal{L}_{\text{gauge}}^{\text{SM}} + \frac{1}{2} (M_R^{(0)})_{ij} \bar{\nu}_R^{ci} \nu_R^j + \text{h.c.} \quad \text{with} \quad (M_R^{(0)})_{ij} = M_\nu \delta_{ij} . \quad (1)$$

The lepton flavour symmetry group of $\mathcal{L}_{(0)}$ is $G_{\text{LF}} = SU(3)_{L_L} \times SU(3)_{e_R} \times O(3)_{\nu_R}$ and the MLFV hypothesis states that G_{LF} is broken only by two irreducible sources, λ_e^{ij} and λ_ν^{ij} , defined by:

$$\mathcal{L}_{\text{Sym.Br.}} \supset -\lambda_e^{ij} \bar{e}_R^i (H^\dagger L_L^j) + i\lambda_\nu^{ij} \bar{\nu}_R^i (H^T \tau_2 L_L^j) + \text{h.c.} \quad (2)$$

where H is the usual SM Higgs doublet. Treating $\lambda_{e,\nu}$ as spurions of G_{LF} , this implies the following transformation properties:

$$\lambda_e \rightarrow V_R \lambda_e V_L^\dagger, \quad \lambda_\nu \rightarrow O_\nu \lambda_\nu V_L^\dagger, \quad (3)$$

with $V_L \in SU(3)_{L_L}$, $V_R \in SU(3)_{e_R}$, and $O_\nu \in O(3)_{\nu_R}$. All the operators of the effective theory should respect a formal invariance under G_{LF} according to these rules.

Although in the effective field theory approach we remain agnostic as to the full structure of $\mathcal{L}_{(0)}$ and $\mathcal{L}_{\text{Sym.Br.}}$ (as well as the mechanism generating M_ν , λ_e , λ_ν), the MLFV postulates, implemented through the spurion technique, provide enough information to address specific questions concerning:

1. *The structure of low energy FCNC couplings.*

The effective coupling governing FCNC transitions of charged leptons to leading order in λ_e , λ_ν , –or the leading $(8, 1, 1)$ spurion of G_{LF} – is [4, 5]

$$\Delta_{\text{FCNC}} = \lambda_\nu^\dagger \lambda_\nu. \quad (4)$$

2. *The structure of ν_R mass splitting.*

The mass degeneracy of the ν_R fields is removed by appropriate combinations of spurions transforming as $(1, 1, 6)$ under G_{LF} . To lowest order in λ_e and λ_ν , we can write

$$M_R = M_R^{(0)} + \sum c_n \delta M_R^{(n)}, \quad (5)$$

where $M_R^{(0)}$ has been defined in Eq. (1) and

$$\begin{aligned} \delta M_R^{(11)} &= M_\nu [\lambda_\nu \lambda_\nu^\dagger + (\lambda_\nu \lambda_\nu^\dagger)^T], \\ \delta M_R^{(21)} &= M_\nu [\lambda_\nu \lambda_\nu^\dagger \lambda_\nu \lambda_\nu^\dagger + (\lambda_\nu \lambda_\nu^\dagger \lambda_\nu \lambda_\nu^\dagger)^T], \\ \delta M_R^{(22)} &= M_\nu [\lambda_\nu \lambda_\nu^\dagger (\lambda_\nu \lambda_\nu^\dagger)^T], \\ \delta M_R^{(23)} &= M_\nu [(\lambda_\nu \lambda_\nu^\dagger)^T \lambda_\nu \lambda_\nu^\dagger], \\ \delta M_R^{(24)} &= M_\nu [\lambda_\nu \lambda_e^\dagger \lambda_e \lambda_\nu^\dagger + (\lambda_\nu \lambda_e^\dagger \lambda_e \lambda_\nu^\dagger)^T], \\ \delta M_R^{(31)} &= \dots \end{aligned} \quad (6)$$

The c_n are arbitrary coefficients whose size depends on dynamical properties: if the Yukawa corrections are generated within a perturbative regime, such as in scenarios of radiative leptogenesis [13], the size of the c_n decreases according to the power of Yukawa insertions (e.g. in a standard loop-expansion one expects $c_{11} \sim g_{\text{eff}}^2/(4\pi)^2$, $c_{2i} \sim c_{11}^2, \dots$). A priori one cannot exclude a strong-interaction regime where all the $c_n \sim \mathcal{O}(1)$. But even in this extreme case, the series in (5) is expected to be dominated by the first few terms if the largest entries of $\lambda_{\nu,e}$ are at most of $\mathcal{O}(1)$ (as assumed in Ref. [4] and expected in most scenarios).

3. *The structure of CP asymmetries in ν_R decays relevant to leptogenesis.*

Denoting by $N_{1,2,3}$ the heavy neutrino mass eigenstates, with masses $M_{1,2,3}$ determined by diagonalization of M_R , the CP asymmetries ϵ_i relevant to leptogenesis are [14, 15, 16]

$$\epsilon_i \equiv \frac{\sum_k [\Gamma(N_i \rightarrow l_k H^*) - \Gamma(N_i \rightarrow \bar{l}_k H)]}{\sum_k [\Gamma(N_i \rightarrow l_k H^*) + \Gamma(N_i \rightarrow \bar{l}_k H)]} = - \sum_{j \neq i} \frac{3}{2} \frac{M_i}{M_j} \frac{\Gamma_j}{M_j} I_j \frac{2S_j + V_j}{3}, \quad (7)$$

where

$$S_j = \frac{M_j^2 (M_j^2 - M_i^2)}{(M_j^2 - M_i^2)^2 + M_i^2 \Gamma_j^2} \quad V_j = 2 \frac{M_j^2}{M_i^2} \left[\left(1 + \frac{M_j^2}{M_i^2} \right) \log \left(1 + \frac{M_i^2}{M_j^2} \right) - 1 \right] \quad (8)$$

and

$$I_j = \frac{\text{Im} \left[(\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger)_{ij}^2 \right]}{|\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger|_{ii} |\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger|_{jj}} \quad \frac{\Gamma_j}{M_j} = \frac{|\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger|_{jj}}{8\pi}. \quad (9)$$

The factors S_j and V_j arise respectively from one-loop self-energy and vertex contribution to the decay widths of the heavy neutrinos (for quasi degenerate right-handed neutrinos, $S \gg V$).

In Eqs. (7)–(9) $\bar{\lambda}_\nu$ indicates the neutrino Yukawa coupling in the basis where M_R is diagonal. Denoting by \bar{U} the unitary matrix that diagonalizes M_R , one has $\bar{\lambda}_\nu = \bar{U} \lambda_\nu$. It is clear that the size of the ϵ_i is determined by the misalignment between M_R and the spurion

$$h_\nu \equiv \lambda_\nu \lambda_\nu^\dagger. \quad (10)$$

The key issue at this point is whether or not this misalignment can be generated if the only sources of flavour breaking are proportional to λ_ν and λ_e . We show below that it is actually possible, even in the limit $\lambda_e \rightarrow 0$.

2.2 Counting and characterizing CP violating phases

The independent CP-violating phases of the model can be characterized in terms of weak-basis invariants, i.e. quantities that are insensitive to changes of basis or re-phasing of the lepton fields. Using the technique of Ref. [17] or Refs. [18, 19, 20], one can easily check that MLFV models contain six independent CPV invariants coming from the Yukawa sector.¹ Even in the more restrictive case of $\lambda_e = 0$ the model contains three independent CPV invariants.

In order to identify the weak-basis invariants, one defines the most general CP transformation as follows [18]:

$$\begin{aligned} \nu_L &\rightarrow U C \nu_L^*, & e_L &\rightarrow U C e_L^*, \\ e_R &\rightarrow V C e_R^*, & \nu_R &\rightarrow W C \nu_R^*, \end{aligned} \quad (11)$$

¹ In a specific underlying model there might be additional flavour-conserving CPV phases –related to couplings of heavy degrees of freedom– on which our effective field theory approach remains agnostic.

where U, V, W are unitary matrices. The above definition corresponds to a combination of CP and the most general flavour transformation that leaves invariant the gauge-kinetic term. In presence of a generic Majorana mass term M_R for ν_R and Yukawa interactions $\lambda_{e,\nu}$, the theory is CP invariant if and only if there exists a set of U, V, W such that:

$$V^\dagger \lambda_e U = \lambda_e^* , \quad (12)$$

$$W^\dagger \lambda_\nu U = \lambda_\nu^* , \quad (13)$$

$$W^T M_R W = -M_R^* . \quad (14)$$

Given this, the simplest necessary conditions for CP invariance can be cast in the following weak-basis invariant form [20]

$$B_1 \equiv \text{Im Tr} \left[h_\nu (M_R^\dagger M_R) M_R^* h_\nu^* M_R \right] = 0 , \quad (15)$$

$$B_2 \equiv \text{Im Tr} \left[h_\nu (M_R^\dagger M_R)^2 M_R^* h_\nu^* M_R \right] = 0 , \quad (16)$$

$$B_3 \equiv \text{Im Tr} \left[h_\nu (M_R^\dagger M_R)^2 M_R^* h_\nu^* M_R (M_R^\dagger M_R) \right] = 0 , \quad (17)$$

where we used the definition of h_ν in Eq. (10) and M_R denotes a generic heavy neutrino mass term. The invariants $B_{1,2,3}$ are independent and one can construct three other independent invariants that explicitly involve λ_e , by replacing $h_\nu \rightarrow h_e \equiv \lambda_\nu \lambda_e^\dagger \lambda_e \lambda_\nu^\dagger$ in the expressions of $B_{1,2,3}$. Moreover, $B_{1,2,3}$ are in direct correspondence with the CP asymmetries ϵ_i relevant for leptogenesis, as can be seen by expressing them in the weak-basis where M_R is diagonal with eigenvalues $M_{1,2,3}$. In this basis, for example, B_1 reads:

$$\begin{aligned} B_1 = & M_1 M_2 (M_2^2 - M_1^2) \text{Im} ((\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger)_{12}^2) + M_1 M_3 (M_3^2 - M_1^2) \text{Im} ((\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger)_{13}^2) \\ & + M_2 M_3 (M_3^2 - M_2^2) \text{Im} ((\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger)_{23}^2) . \end{aligned} \quad (18)$$

Let us now investigate whether the B_i are not vanishing with specific structures of M_R satisfying the MLFV hypothesis:

- If M_R is proportional to the identity, the hermiticity of h_ν and the cyclic property of the trace operation imply that the B_i vanish identically.
- The next step is to break the degeneracy of the heavy neutrinos in a way consistent with the MLFV hypothesis, using the $\delta M_R^{(i)}$ in Eq. (6). If we stop to terms quadratic in the Yukawa couplings, namely $M_R = M_\nu I + c_{11} \delta M_R^{(11)}$, the B_i are still vanishing (again because of the hermiticity of h_ν and the trace properties). This is consistent with the findings of Ref. [11], where only the mass-splitting $\delta M_R^{(11)}$ was considered.
- The vanishing of $B_{1,2,3}$ is finally avoided if any of the quartic terms in Eq. (6) is considered. This can be verified in a number of ways. The simplest method is to use an explicit parameterization of λ_e and λ_ν (as the one given in the next subsection) to evaluate the traces in the expression of $B_{1,2,3}$.

- It is worth stressing that $B_{1,2,3} \neq 0$ even in the limit $\lambda_e \rightarrow 0$. We will show in Sect. 3 that successful leptogenesis can actually be accomplished in this regime. For instance, the structure $M_R = M_\nu I + c_{21} \delta M_R^{(21)}$ is sufficient to generate non-vanishing CP invariants. We conclude that the vanishing of $B_{1,2,3}$ up to $\mathcal{O}(\lambda_\nu^2)$ is just an accidental cancellation. This fact can be understood by noting that in absence of λ_e the simplest non-vanishing invariants have a structure of the type

$$B_0 = \text{Im Tr} \left[(h_\nu)^a (h_\nu^b)^* (h_\nu)^c (h_\nu^d)^* \right] , \quad (19)$$

with (a, b, c, d) non zero and $b \neq d, a \neq c$. This structure appears in the $B_{1,2,3}$ only if M_R is at least quartic in λ_ν .

In conclusion our general analysis of weak-basis invariants² shows that leptogenesis is *at least in principle* possible within MLFV. In the next section we perform a quantitative study of the asymmetries in order to identify the regions of parameter space in which the baryon asymmetry generated via leptogenesis matches the observed value.

2.3 Explicit parametrization of λ_ν

In order to investigate the phenomenological consequences of the MLFV framework in presence of CP violation, it is convenient to choose a particular weak basis and a parameterization of λ_e, λ_ν .

In the basis where the charged-lepton mass matrix is diagonal, we can write

$$m_\nu^{\text{eff}} \equiv v^2 \lambda_\nu^T M_R^{-1} \lambda_\nu = U_{\text{PMNS}}^* m_{\text{diag}} U_{\text{PMNS}}^\dagger \quad (20)$$

where U_{PMNS} is the mixing matrix of the light neutrinos and the corresponding mass eigenvalues are encoded in $m_{\text{diag}} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. In this basis, the most general form of λ_ν is [21]

$$\lambda_\nu = \frac{1}{v} M_R^{1/2} R m_{\text{diag}}^{1/2} U_{\text{PMNS}}^\dagger \xrightarrow{\text{MLFV}} \frac{M_\nu^{1/2}}{v} R m_{\text{diag}}^{1/2} U_{\text{PMNS}}^\dagger \quad (21)$$

where R is a generic complex orthogonal matrix: $R^T R = 1$. As explicitly indicated in the last expression of Eq. (21), within the MLFV framework M_R is to a very good approximation proportional to the identity matrix ($M_R \approx M_\nu \times I$). The small Yukawa-induced mass-splittings, which are crucial for leptogenesis, can be safely neglected in the reconstruction of the see-saw relation.

The matrix R contains six real parameters and can be decomposed as

$$R = O \times H \quad (22)$$

where O is a real orthogonal matrix and H a complex orthogonal and hermitian matrix

$$H^T H = 1 , \quad H^\dagger = H . \quad (23)$$

² Although in our discussion we have used as starting point the well established invariants introduced in Ref. [20], it is possible to reformulate the above argument entirely in terms of λ_ν and λ_e , without any reference to the mass matrix M_R of right-handed neutrinos.

Both O and H can be expressed in terms of 3 independent parameters. Thanks to the $O(3)_{\nu_R}$ invariance –independently of any assumption about CP invariance– within the MLFV framework we can always choose a basis of right-handed fields such that $O = 1$. Our irreducible parameterization of λ_ν is then

$$\lambda_\nu = \frac{M_\nu^{1/2}}{v} H m_{\text{diag}}^{1/2} U_{\text{PMNS}}^\dagger . \quad (24)$$

In the CP limit, $H = I$. The CPV nature of H is transparent in the following parameterization [22],

$$H = e^{i\Phi} = I - \frac{\cosh r - 1}{r^2} \Phi^2 + i \frac{\sinh r}{r} \Phi , \quad \Phi = \begin{pmatrix} 0 & \phi_1 & \phi_2 \\ -\phi_1 & 0 & \phi_3 \\ -\phi_2 & -\phi_3 & 0 \end{pmatrix} , \quad (25)$$

where the ϕ_i are real parameters and $r = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2}$. The CP-conserving case analysed in Ref. [4, 5] is recovered in the limit³ $\phi_i \rightarrow 0$.

Using the parameterization (24), the strength of FCNC couplings reads

$$\Delta_{\text{FCNC}} = \lambda_\nu^\dagger \lambda_\nu = \frac{M_\nu}{v^2} U_{\text{PMNS}} m_{\text{diag}}^{1/2} H^2 m_{\text{diag}}^{1/2} U_{\text{PMNS}}^\dagger \quad (26)$$

and depends not only on the neutrino mass spectrum and mixing angles, but also on the CPV phases appearing in U_{PMNS} and in H . On the other hand, H also controls the CPV phases linked to leptogenesis:

$$h_\nu \equiv \lambda_\nu \lambda_\nu^\dagger = \frac{M_\nu}{v^2} H m_{\text{diag}} H . \quad (27)$$

Eqs. (26) and (27) reveal an interesting connection between the CPV phases relevant to leptogenesis and the effective strength of LFV processes that will be explored in Section 4. Note that even for small ϕ_i the results for Δ_{FCNC} obtained in the CP-conserving case can be a poor approximation, since the hyperbolic functions in H rapidly become large, and even small off-diagonal entries in H can spoil the hierarchical structure of m_{diag} .

Before concluding this section, let us comment on how the CPV invariants B_i discussed in Section 2.2 depend on the CPV phases of H . Expanding to the first non-trivial order in the CPV parameters ϕ_i , the basic invariant B_0 , –computed for $(a, b, c, d) = (1, 2, 2, 1)$ according to Eq. (19)– reads

$$B_0 \propto \phi_1 \phi_2 \phi_3 (m_{\nu_1}^2 - m_{\nu_2}^2) (m_{\nu_1}^2 - m_{\nu_3}^2) (m_{\nu_2}^2 - m_{\nu_3}^2) . \quad (28)$$

As we will discuss in Section 3, this result has phenomenologically interesting consequences: it implies that in limit $\lambda_e \rightarrow 0$, all the three phases of H must be non vanishing in order to generate a sufficient amount of CP violation.

³ In the following we will often refer to the ϕ_i as to the CPV phases of H ; this notation is a bit misleading since the ϕ_i also control the modulus of H .

3 Numerical analysis of the leptogenesis conditions

Following the standard leptogenesis analyses [16], we assume that the observed baryon asymmetry of the Universe, here defined in terms of the baryon, antibaryon and photon number densities,

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10} , \quad (29)$$

entirely originates from a lepton asymmetry, η_L , transferred to the baryon sector via sphaleron processes [7].

The matter-antimatter asymmetries η_L and η_B are connected by an $\mathcal{O}(1)$ factor which depends on the spectrum of the theory. Working in a generic effective field theory approach, we are not able to determine this factor, as well as many other specific details of the model. Our goal is the determination of the main conditions necessary to achieve the correct order of magnitude of η_B . In particular, we assume $\eta_B \approx -\eta_L/2$ as in the SM and consider the range $[3 \times 10^{-10} \lesssim \eta_B \lesssim 9 \times 10^{-10}]$ as phenomenologically acceptable. The baryon asymmetry η_B can be expressed as

$$\eta_B = 0.0096 \sum_i \epsilon_i d_i \quad (30)$$

where d_i are the washout factors, and the ϵ_i are the CP asymmetries defined in Eq. (7). The washout functions d_i can be found solving numerically the full set of Boltzmann equations. According to the recent analysis of Ref. [23], in the case of three quasi degenerate right-handed neutrinos the washout factors are approximately equal and can be expressed as

$$d_i = d(K_1 + K_2 + K_3) \quad d(K) = \frac{2}{K z_B(K)} \left(1 - e^{-\frac{K z_B(K)}{2}} \right) \quad (31)$$

where the K_i are the decay parameters defined as the total decay widths of the ν_R^i normalized to the expansion rate at $T = M_\nu$ and⁴

$$z_B(K) \approx 2 + 4K^{0.13} e^{-\frac{2.5}{K}} . \quad (32)$$

In order to explore the quantitative results of the framework built in the previous section, we have to determine the flavour structure of M_R specifying the coefficients of the spurions which appear in Eq. (6). In most of our numerical studies, we assume the following structure

$$c^{(11)} = c \quad c^{(21)} = c^2 \quad c^{(24)} = c^2, \quad (33)$$

with the other c_n set to zero. This assumption is quite general and presents all the features relevant for our discussion. For $c \ll 1$ we recover the natural expectation of a perturbative regime, while a prototype of a non-perturbative scenario is obtained for $c \rightarrow 1$. We have explicitly checked that the numerical results are essentially unaffected by the

⁴ The results in Eqs. (31)–(32) provide an excellent representation of the fully numerical solutions in the strong washout regime, in which $K_i > 1$ ($i=1,2,3$). We have checked that in our framework this condition is verified in a large fraction of the parameter space.

substitution of $c^{(21)}$ with any $\mathcal{O}(1)$ combination of $\{c^{(21)}, c^{(22)}, c^{(23)}\}$.⁵ The most relevant features of this assumption are illustrated in Fig. 1, where we show for comparison the limiting cases $c^{(24)} = 0$ (i.e. $\lambda_e \rightarrow 0$) and $c^{(11)} = c^{(21)} = c^{(24)} = c$.

A key observation which holds in all cases is that the order of magnitude of the mass splitting is dominated by $\delta M_R^{(11)}$, the first order term in Eq. (6). This naturally yields to the resonance condition [24] which enhances the self-energy contribution:

$$(M_R^j - M_R^i)/M_R^i \sim |\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger|_{jj} \implies S_j \sim \frac{M_j}{\Gamma_j} \gg V_j. \quad (34)$$

As a result, within the MLFV framework the general formula (7) can always be simplified to

$$\epsilon_i \sim - \sum_{j \neq i} \frac{\text{Im} \left[(\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger)_{ij}^2 \right]}{|\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger|_{ii} |\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger|_{jj}}. \quad (35)$$

The dependence of η_B on the parameter c which determines the size of the mass-splittings is illustrated in the left panel of Fig. 1. As can be noted, the dependence is quite mild for $c \in [0.01, 1]$ and employing the natural hierarchy (33). The dependence is stronger for the case $c^{(11)} = c^{(21)} = c^{(24)} = c$, which however is not well motivated for $c \ll 1$. (In Fig. 1, as in the following plots, the parameters which play a minor role in illustrating a specific functional dependence have been fixed to reference intervals specified in the captions).

The dependence of η_B on c_{24} –the coefficient of the mass-splitting containing λ_e – is well illustrated by the right panel of Fig. 1: the dependence is negligible if $M_\nu \gtrsim 10^{11}$ GeV (see for instance the left panel for $M_\nu = 10^{13}$ GeV), while it is crucial for lighter values of M_ν . This fact can easily be understood by noting that the normalization of λ_ν is proportional to $\sqrt{M_\nu}$ [see Eq. (24)]. For sufficiently large values of M_ν we enter the regime where the flavour-violating asymmetry induced by the quartic terms in λ_ν is much larger than the one induced by the mass splitting containing λ_e . This also explains the growth of η_B with M_ν for $M_\nu \gtrsim 10^{11}$ GeV (see appendix). Interestingly, the experimental value of η_B in Eq. (29) indicates that the region of M_ν where the effects of λ_e can be neglected is the phenomenologically relevant one. In particular, varying the free parameters in what we consider to be their natural range (see Fig. 3, upper panel), we find that a baryon asymmetry compatible with experiments implies $M_\nu \gtrsim 10^{12}$ GeV. Since we have not explicitly solved the full set of Boltzmann equations (we relied on the approximate formulae of Ref. [23]), and we have chosen a specific set of free parameters, this bound cannot be taken as a strict lower limit, but it should be interpreted as the natural lower scale for successful leptogenesis in this general framework. Note that, similarly to the negligible influence of λ_e in the initial values of the CPV asymmetries, in the high M_ν region we can neglect other flavour effects which have been addressed in the recent literature [25].

⁵ There is only one pathological choice, namely $c^{(21)} = c^{(22)} = c^{(23)}$. In this case the quartic term in Eq. (6) is the square of the quadratic one and the CPV invariants vanish identically as in the case with no quartic terms (see Sect. 2.2).

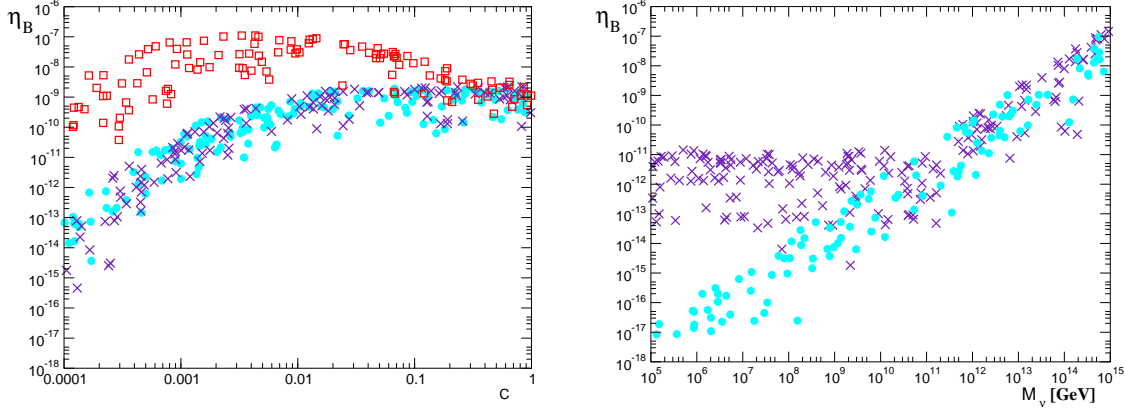


Figure 1: Dependence of η_B on the size of the mass-splitting parameter (c) and on the absolute mass scale (M_ν) of the ν_R mass matrix. *Left panel:* η_B as a function of c . The light blue circles correspond to the reference hierarchy in Eq.(33); the violet crosses are obtained for $c^{(24)} = 0$; the red squares are obtained for $c^{(11)} = c^{(21)} = c^{(24)} = c$; in all cases $M_\nu = 10^{13}$ GeV. *Right panel:* η_B as a function of M_ν . Conventions as in the left panel, with $c \in [0.001, 0.1]$. In both plots $\phi_i \in [0.01, 0.6]$ and $m_\nu \in [10^{-4}, 10^{-2}]$ eV.

The left panel of Fig. 2 illustrates the dependence of η_B on the three CPV parameters ϕ_i contained in the matrix H . In this plot the ϕ_i have been set to the same value. As mentioned at the end of Sect.2.3, in the limit where we can neglect λ_e , the CPV asymmetries depend on the products of the three ϕ_i . For this reason, the maximal effects are obtained when the three ϕ_i are similar in size. The decrease of the baryon asymmetry for larger values is due to a partial cancellation between the two terms in Eq. (35), as consequence of the peculiar properties of the matrix H [see Eq. (23)]. The suppression becomes more effective for large ϕ_i and for a quasi-degenerate spectrum of the light neutrinos, as shown in the right panel of Fig. 2. All plots are obtained assuming a normal hierarchy for the light neutrinos, but we have checked that the results are almost unchanged in the case of inverted hierarchy.

In conclusion, the numerical study shows that the correct order of magnitude of the baryon asymmetry can be obtained in the MLFV framework. The key ingredients are a sufficiently high overall scale of right-handed neutrinos ($M_\nu \gtrsim 10^{12}$ GeV) and sufficiently large CPV parameters in the matrix H ($|\phi_i| \gtrsim 0.01$).

4 Implications for low-energy FCNC transitions

In this section we briefly analyse the consequences on low-energy FCNC rates following from the requirement of successful leptogenesis. In particular, we are interested in understanding: i) what are the implications on the overall rate of FCNC transitions; ii) whether the predictions of ratios such as $\mathcal{B}(\mu \rightarrow e\gamma)/\mathcal{B}(\tau \rightarrow \mu\gamma)$ derived in Ref. [4, 5] in the limit of CP conservation are still valid.

We focus the attention on $l_i \rightarrow l_j\gamma$ processes only. The ratio of the corresponding branching ratios depends uniquely on the U_{PMNS} matrix, the effective light neutrino

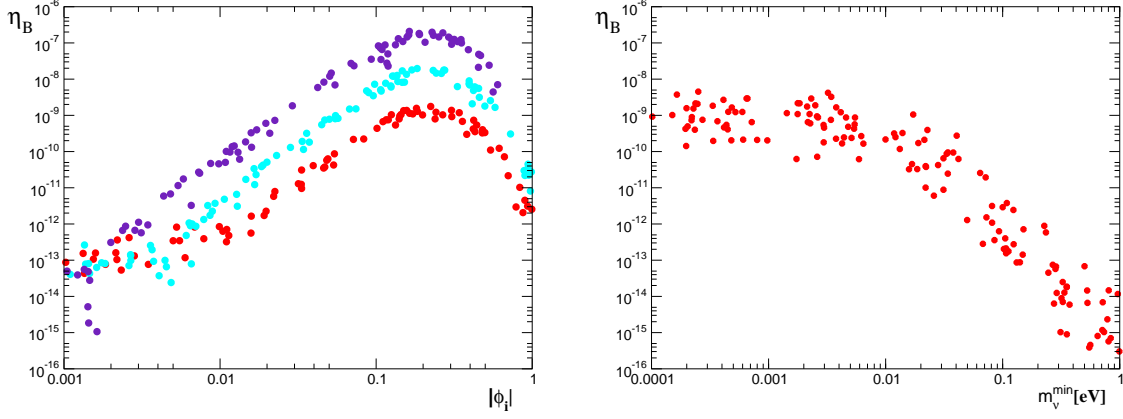


Figure 2: Dependence of η_B on the CPV parameters of λ_ν (ϕ_i) and on the mass of the lightest left-handed neutrino (m_ν^{\min}). *Left panel:* η_B as a function of $\phi = \phi_1 = \phi_2 = \phi_3$ for $M_\nu = 10^{13}$ GeV (red, lower curve), $M_\nu = 10^{14}$ GeV (light blue, middle), $M_\nu = 10^{15}$ GeV (violet, upper), with $m_\nu^{\min} = [10^{-4}, 10^{-2}]$ eV. *Right panel:* dependence of η_B on m_ν^{\min} , for $M_\nu = 10^{13}$ GeV and $\phi_i \in [0.1, 0.6]$ (also the CP asymmetries ϵ_i show a similar dependence on m_ν^{\min}). In both plots $c \in [0.01, 0.1]$. Only the points satisfying $|\lambda_\nu| \lesssim 1$ are plotted.

masses and the leptogenesis phases. Moreover, the absolute rates of these processes are particularly simple –compared to other FCNC transitions– being determined by only two independent dimension-six effective operators. This allows us to assess in cleaner context the impact of the CPV parameters which have been neglected in Ref. [4, 5]. Finally, significant experimental improvements on both $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu(e)\gamma$ are expected in the near future [26]. A detailed study of the dependence of different low-energy observables on the leptogenesis and Majorana phases in a similar context (SUSY see-saw with quasi-degenerate ν_R) has been recently presented in Ref. [27].

In the MLFV framework, the effective Lagrangian relevant for the radiative decays $l_i \rightarrow l_j\gamma$ is

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda_{\text{LFV}}^2} \left(c_{RL}^{(1)} O_{RL}^{(1)} + c_{RL}^{(2)} O_{RL}^{(2)} \right), \quad (36)$$

where

$$\begin{aligned} O_{RL}^{(1)} &= g' H^\dagger \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta_{\text{FCNC}} L_L B_{\mu\nu}, \\ O_{RL}^{(2)} &= g H^\dagger \bar{e}_R \sigma^{\mu\nu} \tau^a \lambda_e \Delta_{\text{FCNC}} L_L W_{\mu\nu}^a, \end{aligned} \quad (37)$$

and g' (g) and $B_{\mu\nu}$ ($W_{\mu\nu}^a$) are the coupling constant and the field strength tensor of the $U(1)_Y$ ($SU(2)_L$) gauge group. This effective Lagrangian leads to [4]

$$\begin{aligned} B_{l_i \rightarrow l_j \gamma} &\equiv \frac{\Gamma(l_i \rightarrow l_j \gamma)}{\Gamma(l_i \rightarrow l_j \nu_i \bar{\nu}_j)} = 384 \pi^2 e^2 \frac{v^4}{\Lambda_{\text{LFV}}^4} \left| (\Delta_{\text{FCNC}})_{ij} \right|^2 |c_{RL}^{(2)} - c_{RL}^{(1)}|^2 \\ &= \left(\frac{v M_\nu}{\Lambda_{\text{LFV}}^2} \right)^2 |c_{RL}^{(2)} - c_{RL}^{(1)}|^2 \hat{f}_{l_i \rightarrow l_j \gamma}(U_{\text{PMNS}}, m_\nu^{\min}, \phi_i). \end{aligned} \quad (38)$$

The coefficients of the operators and the effective new-physics scale Λ_{LFV} (expected to be in the TeV region) are unknown, but they both cancel in the ratios of the various FCNC rates. For simplicity, we will set $|c_{RL}^{(2)} - c_{RL}^{(1)}| = 1$ in the following.

The most significant implication on FCNC rates derived from the requirement of successful leptogenesis is the constraint on the overall normalization of Eq. (38). As we have seen in the previous section, the overall neutrino mass scale M_ν should exceed 10^{12} GeV in order to generate the observed value of the baryon asymmetry (see Fig. 3, upper panel). This breaks the ambiguity in the normalization of λ_ν and consequently on the ratio $(\nu M_\nu)/\Lambda_{\text{LFV}}^2$ appearing in Eq. (38). In the analyses of Ref. [4, 5], the ambiguity on the value of M_ν prevented the extraction of lower bounds on the new-physics scale Λ_{LFV} using the experimental constraints on FCNC rates. This becomes possible imposing the additional requirement of successful leptogenesis.

In the lower panel of Fig. 3 we plot the $\mu \rightarrow e\gamma$ branching ratio as a function of Λ_{LFV} . We generated a large number of events extracting randomly all the parameters in a wide range (see figure caption) and evaluated $\mathcal{B}(\mu \rightarrow e\gamma)$ only for those events yielding the baryon asymmetry within the observed range.⁶ Despite the spread of the points –due to the large number of parameters involved– a clear correlation between the values of the branching ratio and Λ_{LFV} emerges. A similar correlation holds also for $\mathcal{B}(\tau \rightarrow \mu\gamma)$ and for other $\mu \rightarrow e$ transitions, that can be expressed in terms of $\mathcal{B}(\mu \rightarrow e\gamma)$ up to known phase space integrals and unknown ratios of Wilson coefficients [5]. This allows us to identify the ranges of Λ_{LFV} that current and future low-energy LFV experiments can probe. In particular, it is worth stressing that $\mu \rightarrow e\gamma$ is already probing new-physics scales higher than those probed by FCNC transitions in the quark sector [28]. If no signal is observed by the MEG experiment at PSI, scales as high as 100 TeV will be excluded, in contradiction with the natural expectation of the MLFV hypothesis: therefore a definite prediction of this framework is that $\mu \rightarrow e\gamma$ should be observed soon.

The second information that can be extracted from the leptogenesis results is the allowed range of the CP violating parameters ϕ_i appearing in the matrix H in Eq. (25). The impact of these parameters in the predictions of the $\mathcal{B}(\mu \rightarrow e\gamma)/\mathcal{B}(\tau \rightarrow \mu\gamma)$ ratio is illustrated in Fig. 4. In the upper panel we plot $\mathcal{B}(\mu \rightarrow e\gamma)/\mathcal{B}(\tau \rightarrow \mu\gamma)$ as a function of the s_{13} parameter of the U_{PMNS} matrix. We extracted randomly all the parameters contained in λ_ν as in Fig. 3, selecting the events satisfying the leptogenesis constraint. The same quantity is plotted in the left panel of Fig. 4 with the leptogenesis phases set to zero. As can be seen, in general the inclusion of the high-energy phases largely spoils the CP-conserving predictions; however, the pattern $\mathcal{B}(\mu \rightarrow e\gamma) < \mathcal{B}(\tau \rightarrow \mu\gamma)$ is still preserved. Indeed the deviations from the CP conserving case are almost completely driven by the increase of $\mathcal{B}(\mu \rightarrow e\gamma)$, while $\mathcal{B}(\tau \rightarrow \mu\gamma)$ is only marginally affected. This fact can be easily understood: in the CP-conserving limit $(\Delta_{\text{FCNC}})_{\mu e}$ is suppressed by the small values of s_{13} and $\Delta m_\nu^{\text{sol}}$ [4]. Even if not large in magnitude, the flavour off-diagonal CPV parameters of H can spoil these suppression factors yielding to larger values of $\mathcal{B}(\mu \rightarrow e\gamma)$. Their effect is much smaller for $\mathcal{B}(\tau \rightarrow \mu\gamma)$ since $(\Delta_{\text{FCNC}})_{\tau\mu}$ is proportional to the largest neutrino mass splitting $(\Delta m_\nu^{\text{atm}})$ already in the CP-conserving

⁶ Having explicitly checked that the Majorana phases of U_{PMNS} have a negligible impact on the $l_i \rightarrow l_j\gamma$ rates, these phases have been set to zero both in Fig. 3 and in Fig. 4.

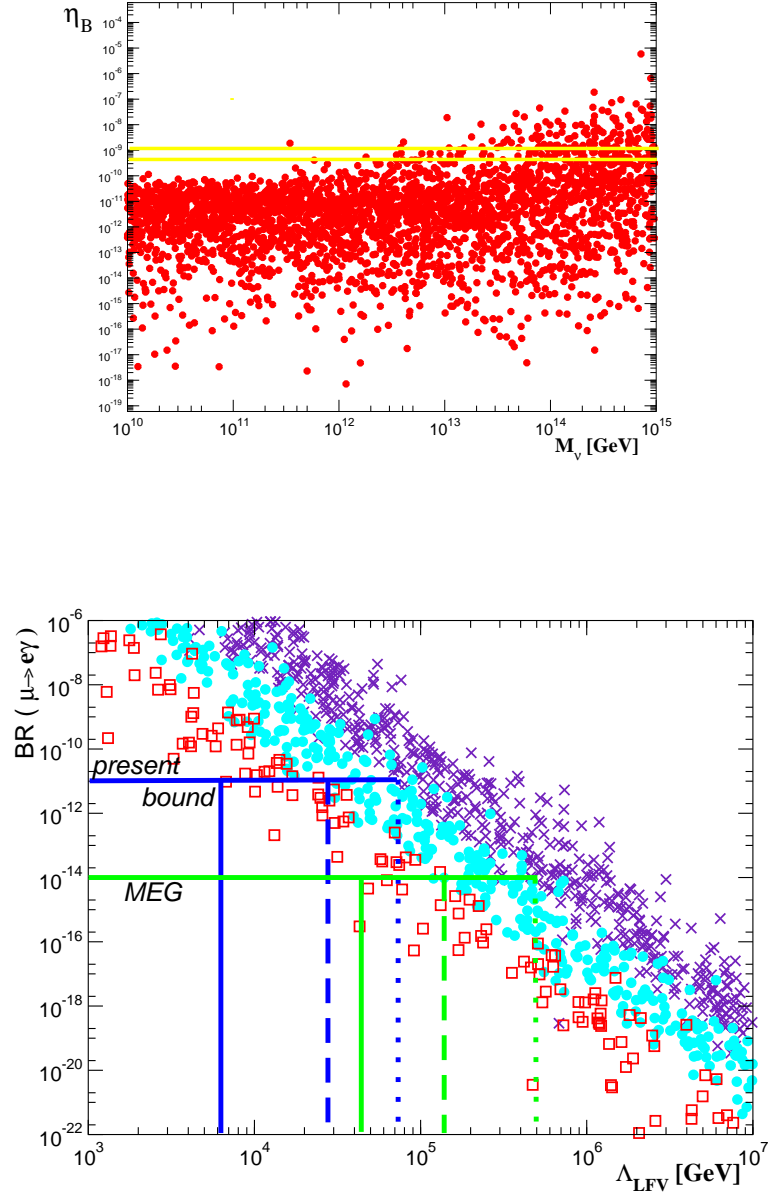


Figure 3: Numerical studies on the scales of lepton number (M_ν) and lepton flavour (Λ_{LFV}) violation. *Upper panel:* η_B as a function of M_ν , varying the other parameters in the following ranges: $c \in [0.001, 1]$, $\phi_i \in [0.001, 1]$ and $m_\nu^{min} \in [10^{-4}, 0.6]$ eV. *Lower panel:* $\mathcal{B}(\mu \rightarrow e\gamma)$ vs. Λ_{LFV} for the points in the yellow band on the upper panel (satisfying the leptogenesis constraint), grouped according to their value of M_ν : violet crosses for $M_\nu > 10^{14}$ GeV, light blue circles for $10^{14} \text{ GeV} > M_\nu > 10^{13}$ GeV, red squares for $M_\nu < 10^{13}$ GeV. The present bounds (future MEG sensitivity) on Λ_{LFV} are shown by the blue (green) lines: full line for $M_\nu = 10^{13}$ GeV, dashed line for $M_\nu = 10^{14}$ GeV, dotted line for $M_\nu = 10^{15}$ GeV. Without further information on M_ν and leptogenesis parameters, the bounds indicated by the full lines should be taken as conservative estimates.

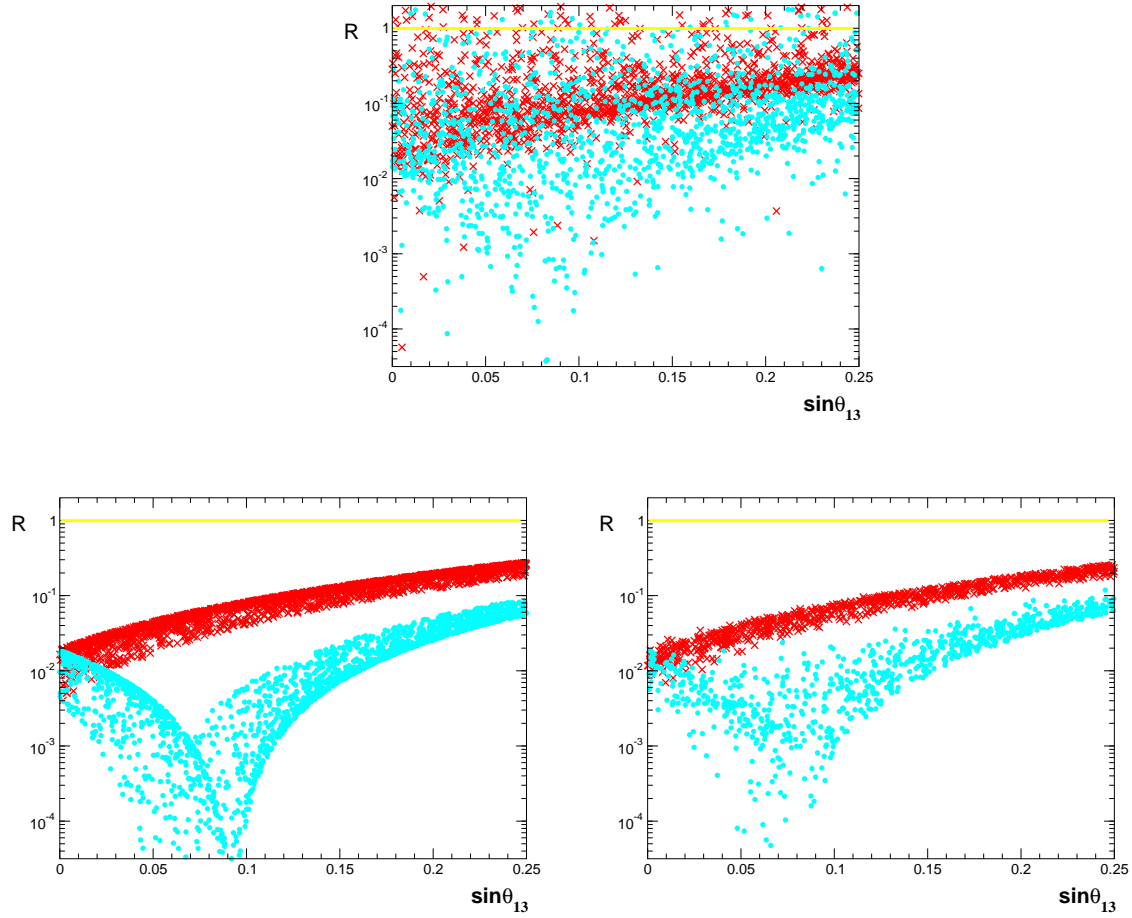


Figure 4: $R = \mathcal{B}(\mu \rightarrow e\gamma)/\mathcal{B}(\tau \rightarrow \mu\gamma)$ for different ranges of the CPV parameters. The light blue circles correspond to $\delta = 0$, the red crosses to $\delta = \pi$. *Upper panel*: general result with CPV phases after imposing the leptogenesis constraint. *Left panel*: no leptogenesis CPV phases. *Lower panel*: CPV case after imposing the leptogenesis constraint, with $M_\nu > 10^{15}$ GeV and $\phi_i < 0.1$. In these plots, $c \in [0.001, 1]$ and $m_\nu^{min} \in [10^{-4}, 0.6]$ eV.

case.

The strong dependence of FCNC amplitudes on the complex part of the matrix R defined in Eq. (21) is an intrinsic property of the see-saw mechanism with quasi-degenerate right-handed neutrinos, and is not specific of the MLFV framework [27]. As far as MLFV models are concerned, we find that there is an interesting regime of small phases where we recover the strong predictive power of the CP-conserving case (see Fig. 4 right panel). This regime holds for high values of M_ν ($M_\nu \sim 10^{15}$ GeV), as shown in the left panel of Fig. 2. Such high values are quite welcome in grand-unified models and give rise to a natural order of magnitude for λ_ν (with maximal eigenvalue of order one, in close analogy with the top-quark Yukawa coupling). On the other hand, high values of M_ν enhance the tension with the lower bound on Λ_{LFV} set by $\mathcal{B}(\mu \rightarrow e\gamma)$. Besides these theoretical considerations, this regime is particularly interesting from a pure phenomenological point of view: the comparison of the precise predictions of FCNC ratios with experimental data provides a powerful testing tool.

5 Conclusions

Within the quark sector the MFV hypothesis provides a natural solution to the flavour problem: it is quite impressive that the huge amount of experimental data available on quark-flavour mixing is compatible with the hypothesis that the Yukawa couplings are the only sources of flavour symmetry breaking [28]. If there is a deep dynamical reason behind this phenomenological observation, it is natural to expect a similar mechanism at work –at least to some extent– also in the lepton sector.

In this work we have analysed the phenomenon of CP violation in the generic class of models with three right-handed neutrinos satisfying the criterion of MLFV (with extended field content) proposed in Ref. [4]: a global lepton-flavour symmetry $SU(3)_{L_L} \times SU(3)_{e_R} \times O(3)_{\nu_R}$ broken only by two irreducible Yukawa structures $\lambda_e \sim (\bar{3}, 3, 1)$ and $\lambda_\nu \sim (\bar{3}, 1, 3)$. Explicit realizations of this general scenario can be implemented in the minimal supersymmetric SM (MSSM) with see-saw mechanism (see e.g. Ref. [29]), or in models with TeV-scale neutrinos, such as those analysed in Ref. [11, 24]. We have shown that in this class of models it is possible to generate a phenomenologically acceptable leptogenesis, satisfying the MLFV hypothesis, only if the overall scale of right-handed neutrinos is sufficiently high: $M_\nu \gtrsim 10^{12}$ GeV.

This result was not trivial a priori given the strong restrictions on the model imposed by the flavour symmetry, which in first approximation implies degenerate right-handed neutrinos. By means of a general analysis of the CPV invariants, we have demonstrated that the theory possesses enough physical CPV parameters contributing to leptogenesis, even in the limit $\lambda_e \rightarrow 0$. As far as the parameterization of λ_ν is concerned, it turns out that in this class of models λ_ν can be unambiguously expressed in terms of 9 low-energy parameters (3 light-neutrino mass eigenvalues and 6 parameters of U_{PMNS}), one overall scale (M_ν), and only 3 high-energy CPV parameters (the remnants of the matrix elements of the Casas-Ibarra matrix R [21], after imposing the $O(3)_{\nu_R}$ invariance). The latter are responsible for leptogenesis.

From a dynamical point of view, the mechanism which allows leptogenesis in this

framework is the breaking of the exact ν_R degeneracy by means of Yukawa-induced corrections. This mechanism is necessarily present in any underlying theory because of radiative corrections. The MLFV hypothesis forces it to be the only source of breaking of the ν_R degeneracy. This well-defined pattern for the breaking of the ν_R degeneracy is the origin of the interesting constraints on M_ν derived in this framework and, more generally, of the good predictive power of this class of models. We stress that this appealing feature is not present in generic models with quasi-degenerate heavy neutrinos. The $M_\nu \gtrsim 10^{12}$ GeV bound we have derived is the result of a numerical scan with approximate solutions to the Boltzmann equations, therefore it cannot be considered as a strict lower limit. Rather, it should be regarded as the natural lower scale for successful leptogenesis in this generic framework. In the appendix we have shown how a semi-quantitative understanding of this lower limit can be obtained in terms of simple analytical formulae.

The consequences on low-energy FCNC rates following from the requirement of successful leptogenesis have been investigated, with particular attention to the $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ processes. The most striking consequence of this additional requirement is the lower bound on M_ν , which breaks the ambiguity in the normalization of λ_ν and thus of the FCNC rates. As a result of the lower bound on M_ν , for natural values of Λ_{LFV} in the TeV range (i.e. assuming new particles carrying lepton flavour at the TeV scale, such as for instance within the MSSM), the $\mu \rightarrow e\gamma$ rate turns out to be quite close to its present exclusion limit, and well within the reach of the MEG experiment. For the same reason, the rates for $\mu \rightarrow e\bar{e}e$ and $\mu \rightarrow e$ conversion in nuclei are expected to be one or two orders of magnitude below the present experimental limits.

On general grounds, if the three high-energy CPV parameters are not vanishing the predictions for low-energy FCNC transitions derived in Ref. [4] in the CP-conserving limit can be substantially modified. After imposing the leptogenesis conditions, we find that $\mathcal{B}(\mu \rightarrow e\gamma)$ can easily be enhanced with respect to the CP-conserving case, while $\mathcal{B}(\tau \rightarrow \mu\gamma)$ is quite stable. As a result, in general the specific pattern $\mathcal{B}(\mu \rightarrow e\gamma) \ll \mathcal{B}(\tau \rightarrow \mu\gamma)$ [4] does not hold, but only the weaker condition $\mathcal{B}(\mu \rightarrow e\gamma) < \mathcal{B}(\tau \rightarrow \mu\gamma)$ is satisfied. Interestingly, we have also found that there exists a region of the parameter space where leptogenesis condition can be accomplished with very small CPV parameters, such that the full predictive pattern of FCNC transitions holds as in the CP-conserving case.

Acknowledgment

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Note Added

During the refereeing process of this article, a preprint addressing the viability of leptogenesis in the MLFV framework has appeared [30]. The analysis of Ref. [30] confirms

our main result about the viability of leptogenesis in MLFV via Yukawa-induced corrections, and all our analytical treatment for the inclusion of CPV effects in this framework. As far as the estimate of η_B is concerned, the qualitative behavior with almost no dependence for $M_\nu \lesssim 10^{11}$ GeV and linear growth for $M_\nu \gtrsim 10^{11}$ GeV is also confirmed. However, the authors of Ref. [30] claim that a more refined treatment of Boltzmann equations with respect to Ref. [23] (i.e. the approach we have adopted) and the inclusion of flavour-dependent effects, raises the maximal values of $\eta_B(M_\nu)$ slightly above the 10^{-10} level also in the low- M_ν region. If confirmed, this result would weaken our conclusion about the lower bound on M_ν . However, we stress that even in the approach of Ref. [30] only a marginal region of the parameter space can account for the experimental value of η_B for $M_\nu \lesssim 10^{12}$ GeV. We still expect that the conclusions reached in the present work by means of a less sophisticated analysis of leptogenesis represent the basic features expected in most models with MLFV.

A On the scaling of η_B with M_ν

We present here a simplified discussion to illustrate the scaling of η_B with M_ν (and the resulting lower bound on M_ν) in a simple analytical form. In particular, we prove that in the high- M_ν regime (where λ_e terms can be neglected) $\eta_B = \kappa M_\nu$ and we provide a reasonable estimate of the dimensional parameter κ .

Let us start from the formula for the baryon asymmetry:

$$\eta_B = 9.6 \cdot 10^{-3} \times d(K_1 + K_2 + K_3) \times (\epsilon_1 + \epsilon_2 + \epsilon_3) . \quad (39)$$

The dilution factor $d(K_1 + K_2 + K_3)$ does not depend on M_ν . The M_ν dependence comes from the asymmetries ϵ_i , which near resonance are described by Eq. (35). In order to identify the scaling of the ϵ_i with M_ν , we count the powers of λ_ν insertions and use the relation $\lambda_\nu \sim M_\nu^{1/2}$ (see Eq. (21)).

The denominator in Eq. (35) scale as $\lambda_\nu^4 \propto M_\nu^2$. The analysis of the imaginary part in the numerator is slightly more involved. In general

$$\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger = \bar{U} \lambda_\nu \lambda_\nu^\dagger \bar{U}^\dagger , \quad (40)$$

where \bar{U} is the unitary matrix which diagonalizes the right-handed mass matrix:

$$\bar{U}^T M_R \bar{U} = M_R^{\text{diag}} . \quad (41)$$

Focusing on a specific structure of M_R with only c_{11} and c_{21} non-zero, we have:

$$M_\nu^{-1} M_R = I + c_{11} \delta_1 + c_{21} \delta_2 \quad \delta_1 = h_\nu + h_\nu^T \quad \delta_2 = h_\nu^2 + (h_\nu^T)^2 \quad (42)$$

where, as usual, $h_\nu = \lambda_\nu \lambda_\nu^\dagger$. Let us also assume the perturbative behavior $c_{11} \sim c$ and $c_{21} \sim c^2$ with $c < 1$, which allows us to perform a perturbative diagonalization of M_R . In general, we can write $\bar{U} = O_1 O_2$, where $O_{1,2}$ are real-orthogonal matrices and O_1 is the matrix diagonalizing δ_1 :

$$M_\nu^{-1} O_1^T M_R O_1 = \bar{M} + c_{21} \bar{\delta}_2 , \quad \bar{\delta}_2 = O_1^T \delta_2 O_1 \quad (43)$$

(with \bar{M} diagonal). As can be explicitly verified in perturbation theory, $O_1 \sim O(1)$ and in first approximation does not depend on M_ν . Now we can proceed with the diagonalization of $O_1^T M_R O_1$ by means of O_2 . To first non-trivial order in $c_{22}\bar{\delta}_2$ we can write $O_2 \approx I + A$, with

$$A^{ij} = \frac{c_{21}\bar{\delta}_2^{ij}}{\bar{M}^{jj} - \bar{M}^{ii}} \propto \frac{h_\nu^2}{h_\nu} \sim \lambda_\nu^2 \propto M_\nu . \quad (44)$$

Given the quadratic dependence on h_ν in the numerator, in this case we get a non-trivial scaling with M_ν . Putting together the above results and recalling that

$$\text{Im} [(O_1 \lambda_\nu \lambda_\nu^\dagger O_1^T)_{ij} (O_1 \lambda_\nu \lambda_\nu^\dagger O_1^T)_{ij}] = 0 \quad (45)$$

(see sect. 2.2), the leading contributions to the asymmetry arise from terms of the type

$$\text{Im} [(\bar{\lambda}_\nu \bar{\lambda}_\nu^\dagger)_{ij}^2] \propto \text{Im} [(A O_1 \lambda_\nu \lambda_\nu^\dagger O_1^T)_{ij} (O_1 \lambda_\nu \lambda_\nu^\dagger O_1^T)_{ij}] \sim \lambda_\nu^6 \propto M_\nu^3 . \quad (46)$$

We are now finally able to evaluate the complete scaling of the baryon asymmetry. As expected, η_B grows linearly with M_ν . More explicitly, we find

$$\eta_B \sim 9.6 \cdot 10^{-3} \times d \times \frac{\sqrt{\Delta m_{\text{atm}}^2}}{v^2} \times f(\phi_1, \phi_2, \phi_3) \times M_\nu , \quad (47)$$

where $f(\phi_1, \phi_2, \phi_3) \sim \phi_1 \phi_2 \phi_3$ up to higher orders in the phases. Numerically, using $d \sim 10^{-3}$ and $f(\phi_1, \phi_2, \phi_3) \sim O(10^{-1})$ (moderately large CPV phases) we get

$$\eta_B \sim 1.2 \times 10^{-21} \times \frac{M_\nu}{\text{GeV}} . \quad (48)$$

Requiring $\eta_B > \eta_B^{\text{exp}} = 6.3 \times 10^{-10}$ implies $M_\nu > 5 \times 10^{11}$ GeV, in remarkable agreement with our findings from the numerical scan.

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